Teaching statement

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I am fortunate to have spent my graduate career at the University of Arizona, where I have had the opportunity to develop opinions about teaching by having had full responsibility for the classroom. Having spent a decade in industry before returning for graduate school, I bring to the classroom a hard-earned appreciation for problem-solving strategies and error detection. The techniques of (a) knowing what to do when we don't know what to do and (b) finding the (inevitable) errors in our work as soon as we make them are the most essential things I have to give my students — with value far beyond the mathematics classroom. In this note, I discuss how organization and communication facilitate development of these crucial life skills.

Organization: Good planning goes unnoticed; lack of planning is obvious to everyone in the classroom. I prefer a conversational, extemporaneous mode of speaking, so I prepare detailed lesson plans, focusing in particular on topics to be covered and computational details. Most of the time, having used preparation time to mentally load the material, I consult my lesson plan only for occasional guidance.

Participation: I have been told, by students as well as peers, that my speaking is engaging, personable, and informative. Yet, in teaching, this is not enough: regardless of the quality of the instructor's performance, the student is nonetheless passively viewing it. Effective teaching requires student involvement. For this to happen, students need to feel comfortable in the classroom.

Every semester, I emphasize the difference between passive learning (*That makes sense*) vs. active learning (*. . . Wait, how do I do this again?*). The most important things happen when I am *not* talking: when I've asked students a question and I am waiting for an answer, when students are working problems, when they are collaborating with one another, etc.

There are two main challenges in classroom instruction: (a) few of us want to say something wrong (or questionable) in front of our peers, and (b) the students who know what they're doing tend to speak first. To address the former problem, I always try to catch something positive about what a student contributes. I take the same approach with grading: instead of an X through their arithmetic error in red pen, a smiley face in green ink is an acknowledgement that we all make mistakes. For the latter problem, I make it clear that we are not having a horse race to the first correct answer. I tell them we just saw the concept 5 minutes ago, so I don't expect mastery — I just want to get a feel for where they are. I pitch this as a learning experience *for me*. I turn it around: I give them a problem and ask them to tell me what's hard, what's keeping them from solving the problem, if they're stuck at a certain step, if they don't know how to start, etc. When we are focusing on the thought process itself, rather than its final result, we are truly learning.

Calculators: As a technophile, I appreciate the availability of computers and calculators: using them is far more pleasant than having to graph everything by hand, as I did when I was young. Also, I admit to myself that most of my students won't be using mathematics daily for the rest of their lives, but most of them will be using technology daily.

I emphasize using calculators for confirmation: if we do the algebra to find the zeroes of a function, and if that also matches with what we see graphically on the calculator, we can have great confidence in our work. At the same time, I emphasize that calculators are not enough. For example, the graph of $y = \sqrt{1 - x^2}$ doesn't touch the x axis — yet this does not mean it has no x-intercepts. Moreover, a calculator can show us a turning point of $y = 1/(1 + x^2)$ — but we can use calculus to find a turning point of $y = a/(b + x^2)$, and moreover prove that this is the *only* turning point. An analytical solution can be more insightful — and in many advanced cases, *quicker* to obtain — than a numerical solution.

Homework: Grading of quizzes and homework, along with office hours and tutoring, is for me the heart of teaching. Here I have the opportunity to see what my students are really doing. Yet, nothing is as time-consuming as grading. In my most recent teaching semester, I used the on-line homework system WebAssign in combination with traditional written homework. The results were mixed: students hated it at first, due to the unfamiliar interface; eventually, most of them preferred it for its immediate feedback. I used WebAssign for drill-type problems, e.g. finding the derivative of 20 different functions. For such tasks, WebAssign was a quicker grader than I am. For thought problems, it was of course not relevant. I used written work for one of every three or four assignments. This enabled me to strike a balance between, on the one hand, freeing time for non-grading tasks, and on the other hand seeing my students' thoughts on paper.

Office hours and tutor center: This is where I believe the most effective learning happens. We all have students who were already excellent on day one of the class; we cannot take credit for their success. But the students who work hard in my classes and truly *improve* over the semester are always the ones who consistently come into my office for help.

As I noted above, most of us are uncomfortable saying something wrong in front of a room of our peers. One on one, students will come out and say, *What I really don't get is . . .* , and then we can get some work done. I value these interactions, not only because the student is able to locate their trouble spot, and not only because the conversational feedback loop is immediate and productive, but because this is my best opportunity to learn how they are thinking about mathematics. Operating on the almost-always-correct hypothesis that one student is brave enough to ask a question that ten others are keeping to themselves, I regularly use these office-hour experiences to guide my lecture the next day.

One of my most vivid experiences in the departmental tutoring room involved a student who asked for help in completing the square. Had I been in a hurry, I might have pointed her to the relevant section in the textbook and moved on. Instead, I resolved to find the root cause. After a few minutes of interaction, she said she *did* know how to complete the square — she could do another problem but not this one. So, I asked for her patience, and asked her to show me how she did the previous problem; then, I asked her to re-work the second problem as far as she could. Only watching her work, expanding the scope of where she believed her question to be, did we discover the hang-up: the first problem had an integer leading coefficient (2) while the second had a fractional leading coefficient (1/4).

Her problem was solved, not by review of the current-semester method of completing the square, but by review of the prerequisite technique of dividing by a fraction.

Problem solving: One type of understanding is *analytic*: given some information, we systematically follow a process to obtain a result — be it the quadratic formula, the Gram-Schmidt orthonormalization procedure, or Buchberger's algorithm. These skills may be easy or difficult, but once mastered, *there is no doubt what to do next.* The other kind of understanding is *synthetic*: given some information, we must free-associate for possible solutions. These questions are much harder because they depend on the *simultaneous, parallel* recall of many, many facts. (And of course, the real-world problems we get paid well to work on are all of this latter type!) There is no clear indication of what to do next — we must find our own way. To a learner, this smells like black magic: how could they have known to pull that particular rabbit out of the hat at that particular time?

At Arizona, we emphasize the Rule of Four: concepts should be presented graphically, algebraically, numerically, and verbally. When I write an exam, I include questions in each of these four formats: this allows students to show me the ways in which they understand a concept. This approach acknowledges the diversity of learning psychologies we all have.

For problem-solving, I also use the Rule of Four: for example, try drawing a picture whenever we're stumped by the symbols. Or, when we don't know what equations to write down, we can first write down in words precisely what it is we don't know. I like to say, *When you are faced with a question you can't answer, it might be the question's fault rather than your own. Keep improving the question until it answers itself.*

When I give students some tabular data and ask them to fit a formula (an example of a synthetic problem), I find that many of them will assume a linear fit. To address this problem, I suggest that they first graph the data, and let the *data* tell *them* what it looks like. Alternatively, by numerically computing adjacent differences and adjacent ratios of tabular data, we can see if a linear or exponential model is appropriate to even *try* using, before delving into the algebraic details of solving for parameters.

For first-semester calculus at Arizona, I have taught from the reform text of Hughes-Hallett et al. My experiences were positive: before drilling in mechanical rules such as that the derivative of x^2 is $2x$, we spent adequate time learning how derivatives and integrals should behave, and how to interpret them.

Hand-in-hand with any detailed work is the fact that we make mistakes. We drop minus signs, we turn z's into twos and sixes into zeroes, and so on, and we will keep doing so all our lives. I always tell my students: plan to make mistakes. I found in my industrial career that mistakes become exponentially more expensive to fix the longer the time from their creation to their detection: when I make a mistake at my desk and catch it five minutes later, no one is the wiser. If I hurry and don't check my work, and the error propagates into an integrated software build, I've inconvenienced perhaps several test personnel. If they don't catch the mistake either, the result can include end-user down time, on-site updates, and customer dissatisfaction. The same will hold true in students' future careers, whatever those may be, and the same holds true in math. We can take a few seconds to double-check each line of our algebra, instead of having to restart a full page of computation; we can check the units of our work using dimensional analysis. We can sanity-check our numbers to make sure they're in range; we can compare analytical and graphical/numerical results.

See also `http://math.arizona.edu/~kerl/teaching/prodev/597T-11.pdf` for a sample lesson plan which uses these concepts.