HOW TO SOLVE THE RUBIK'S CUBE

1. Introduction

The following is an algorithm for solving the Rubik's Cube. The algorithm is not my own; rather, it is one I memorized from a book I read (and subsequently lost) twenty years ago. I don't remember either the author's name or the title, so I can't attribute it properly.

(Note June 2008: The book was *The Simple Solution to Rubik's Cube* by James G. Nourse, Bantam, 1981.)

Some of the (very few) group-theoretical observations come from David Singmaster's (apparently self-published) 1980 pamphlet *Notes on Rubik's Magic Cube.* Singmaster's pamphlet contains a different algorithm for solving the cube.

Most of this document describes notation, provides context, etc. The actual number of moves that must be memorized is quite small. Using this algorithm, after some practice you can expect solution times of 2-3 minutes.

John Kerl, Oct. 2003

2. Notation

- **Faces**: Pick a face of the cube; consider this to be the top face. Of the remaining four side faces, pick one to be the front. In this orientation, the faces may be labeled U, D, F, B, L and R, for up, down, front, back, left and right.
- **Pieces**: Edge and corner pieces will be named by pairs and triples of letters, respectively, e.g. UF for the upper front edge and UFR for the upper front right corner.
- **Turns**: Denote a clockwise (as viewed from outside the cube) quarter-turn of a given face by its letter, in boldface. For example, F is the front face; **F** is a clockwise quarter-turn of the front face. Then a half turn of the face will be denoted by the exponent 2, e.g. $\mathbf{F}^2$, and a counterclockwise quarter turn will be denoted by an apostrophe, e.g $\mathbf{F}'$. (Mathematically it would be more appropriate to use an exponent of -1, but the little apostrophes are less obtrusive.)
- **Processes**: A process is nothing more than a sequence of moves applied left to right. A process will be denoted by the string of letters and their associated exponents, e.g. $\mathbf{UF^2R'}$ means turn the upper face a quarter-turn clockwise, then turn the front face a half turn, then turn the right face a quarter-turn counterclockwise. Spaces or dots have no significance other than to separate mnemonic units.

  Also note that U will be U for the entire duration of the solution. However, F will change as you rotate the cube.

3. Overview

The general idea is:

- The center pieces are fixed via internal posts.
- Put the top edges in their correct positions and orientations.
- Put the top corners in their correct positions and orientations.
- Make sure side centers are correct.
- Put the four middle-row edge pieces in the correct positions, with the correct orientations.
- Put the four bottom corner pieces in the correct positions.
- Put the four bottom corner pieces in the correct orientations.
- Put the four bottom edge pieces in the correct positions.
- Put the four bottom edge pieces in the correct orientations.

At each step, you put another 4 pieces into place, while preserving all the pieces already done. The entire top face is easy to do, since you have so many degrees of freedom — so easy, in fact, I won't bother to write it down. The processes get more and more complicated as you go along, since you have more solved pieces to keep in place.

4. The constructible group vs. the solvable group

This section is optional, but do read its last two paragraphs.

If you take apart the cube and randomly assemble it, you will get a permuation on the positions and orientations of the edge and corner pieces. Corners can only go to corners and edges can only go to edges, and centers are fixed. Since there are 12 edges with 2 orientations each and 8 corners with 3 orientations each, there is a total of

$$12!2^{12}\ 8!3^{8} = 2^{29}3^{15}5^{3}7^{2}11 = 519,024,039,293,878,272,000$$

(519 quintillion) possible constructions, called the *constructible group*.

However, if you try to solve a randomly reassembled cube using the following algorithm (or any other) you will probably fail. It can be shown that the set of permutations of the cube obtainable by starting at the solved cube and turning the faces (the *solvable group*) is smaller than the constructible group by a factor of 12:

- Permutations of edge and/or corner pieces must be even: Unless your cube has been taken apart and reassembled incorrectly, you'll never see the cube all solved except for two interchanged edges, or two interchanged corners. You can have two pairs of edges interchanged, or three edges cyclically permuted. You can have two pairs of corners interchanged, or three edges cyclically permuted. You can have a pair of edges interchanged and a pair of corners interchanged.
- Flips of edge pieces (in the correct position, but with orientation changed) must have even parity, i.e. sum to zero mod 2. You won't see the cube solved except for a flipped edge; flipped edges come in pairs.
- Spins of corner pieces (in the correct position, but with orientation changed) must sum to zero mod 3. You won't see the cube solved except for a spun corner, nor two corners both

spun clockwise. If one is spun clockwise, another will be spun counterclockwise. Or, you may have three corners all spun clockwise (or counterclockwise).

That is, the solvable group has index 12 in the constructible group, and has order

$$12!2^1 28!3^8/12 = 2^{27}3^{14}5^3 7^2 11 = 43,252,003,274,489,856,000$$

The *sticker group*, the set of patterns obtainable by peeling off the stickers and replacing them, is astronomically bigger than the constructible group; your chances of solving such a cube are next to zero. Moral: If a toddler has been playing with your cube, please disassemble it and reassemble it first before attempting to solve it by moves on the faces alone.

It may seem odd for me to discuss the things that *can't* happen, but these facts (for me) serve as mnemonic devices to help remember the choices in the algorithm below. (Also, if you *do* encounter one of these forbidden configurations while solving your cube, now you'll know to take it apart and put it back together into a solvable configuration.)

5. Top edges

First, from a scrambled state, put the upper edges in place. Make sure the side color of each upper edge piece matches the color of the center piece on that side — that is, put each upper edge in the correct position, not merely somewhere on top of the cube.

6. Top corners

Second, preserving upper edges, put the upper corners in place.

Perhaps obvious but I'll point it out anyway: Make sure the edge and corner pieces have not only their top color matching the top face of the cube, but with their side colors matching the corresponding center side pieces. That is, getting "all one color" on top does *not* mean the top is solved. Rather, once the top is solved you must have three same-color squares along the top row of each side.

7. Middle centers

Now make sure side centers are correct, e.g. if the top row of the front face is white, turn the remaining two rows of the cube until the white center is in front.

8. Middle edges

There are four possibilities for the edge piece that belongs in FR:

(1) It is already there, with the correct orientation.
(2) It is already there, with the incorrect orientation. Use either one of the following two processes to put another piece there, which will push it down to the bottom, then apply the appropriate one of the following two processes.

(3) The piece that belongs in FR is on the bottom row, with its R color visible from the side of the cube: Turn D to put this piece onto the R face, then apply:

$$\mathbf{DFD'F' \cdot D'R'DR}$$

(4) The piece that belongs in FR is on the bottom row, with its F color visible from the side of the cube: Turn D to put this piece onto the F face, then apply:

$$\mathbf{D'R'DR \cdot DFD'F'}$$

Do the above for all four middle-row edge pieces. Already, your cube is starting to look nice — the top is one solid color, and the four sides are 2/3 done.

9. Bottom corner positions

There are three possibilities. Rotate the D face until one of them is true.

(1) All four corners are in the correct position. Proceed to orient them below.
(2) Two corners are in the correct position, and the two which are interchanged are *not* diagonally opposite one another. Turn the *entire cube* until the two mispositioned bottom corners are at FLD and FRD, then apply:

$$\mathbf{R'D'R \cdot FDF' \cdot R'DR \cdot D^2}$$

(3) Two corners are in the correct position, and the two which are interchanged are diagonally opposite one another. Apply the above process (it doesn't matter which face is front), then turn the cube 180° (so U is still U, but F moves to B), and apply the same process again, then rotate D until all four corner pieces are in their correct positions.

10. Bottom corner orientations

There are several possibilities, and conceivably there are specific processes to handle each. In an attempt to minimize the number of processes to memorize, though, the author of the book I read suggested repeated applications of a single process which spins three corner pieces. You may need to apply this several times. Additionally, there is a process (from Singmaster) which spins a pair of corners.

(1) To spin FRD, BRD and BLD counterclockwise while keeping FLD fixed, apply

$$\mathbf{R'D'RD' \cdot R'D^2RD^2}$$

(2) To spin FRD, BRD and BLD clockwise while keeping FLD fixed, apply

$$\mathbf{D^2R'D^2R \cdot DR'DR}$$

(3) To spin FLD counterclockwise and BRD clockwise, apply:

$$\mathbf{BL'U^2 \cdot LB'D^2 \cdot BL'U^2 \cdot LB'D^2}$$

(4) To spin FLD clockwise and BRD counterclockwise, apply:

$$\mathbf{D^2BL' \cdot U^2LB' \cdot D^2BL' \cdot U^2LB'}$$

Keep applying the above until you get the bottom corners oriented. It will take at most a few iterations.

Now your cube is starting to look *really* good. There are at most four edge pieces jumbled, all on the bottom of the cube.

11. Bottom edge positions

(1) To cycle RD, BD and LD clockwise while keeping FD fixed, apply

$$\mathbf{L'RF \cdot LR'D^2 \cdot L'RF \cdot LR'}$$

(2) To cycle RD, BD and LD counterclockwise while keeping FD fixed, apply

$$\mathbf{L'RF' \cdot LR'D^2 \cdot L'RF' \cdot LR'}$$

If all four edges are out of place, apply either one of the above to get at least one edge in place. Rotate the cube to put that edge in front. Then, apply the appropriate one of the above.

12. Bottom edge orientations

There are now two flipped edges directly across from one another, two flipped edges not directly across from one another, or four flipped edges.

(1) If all four flipped edges are flipped, pick either of the following processes and do it twice (rotating the entire cube of course in between iterations, to flip one pair, then the other pair).

(2) To flip FD and BD, apply

$$\begin{array}{lcl} \mathbf{L'RF} & \cdot & \mathbf{LR'D} \\ \mathbf{L'RF} & \cdot & \mathbf{LR'D} \\ \mathbf{L'RF^2} & \cdot & \mathbf{LR'D} \\ \mathbf{L'RF} & \cdot & \mathbf{LR'D} \\ \mathbf{L'RF} & \cdot & \mathbf{LR'D^2} \end{array}$$

(3) To flip LD and BD, apply

$$\begin{array}{lcl} \mathbf{L'RF} & \cdot & \mathbf{LR'D'} \\ \mathbf{L'RF'} & \cdot & \mathbf{LR'D'} \\ \mathbf{L'RF^2} & \cdot & \mathbf{LR'} \end{array}$$

then turn the cube so R becomes F and apply

$$\mathbf{L'RF \cdot LR'D^2 \cdot L'RF \cdot LR'}$$

13. You are done!!!