

Numerical methods for random spatial permutations

Abstract: Models of random spatial permutations arise in the study of Bose-Einstein condensation. Namely, permutations of sites occur with probabilities depending on lengths of permutation jumps, as well as on interactions between jumps. Below a critical temperature, one observes the onset of long permutation cycles in spite of short-distance permutation-jump interactions. Following work of Betz, Ueltschi, Gandolfo, and Ruiz [1, 2, 3, 4], we present conjectures for some of these models on the cubic unit lattice, along with results obtained by Markov chain Monte Carlo simulations and finite-size scaling. These conjectures include the shift in critical temperature as a function of interaction strength, and statistics of maximum cycle length in comparison to uniform random permutations. We compare our findings to analytical results which are known for related models. We include algorithmic approaches to determination of winding numbers when periodic boundary conditions are placed on the lattice.

References

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