

QUADRATIC AND CUBIC FUNCTIONS DEFINED ON THE
POSITIVE INTEGERS

by
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THE UNIVERSITY OF ARIZONA
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Quadratic and cubic functions defined on the positive integers
and recommend that it be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

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SIGNED: _____ STUART DENT _____

DEDICATION

For my mom.

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LIST OF TABLES

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CHAPTER 1

CHAPTER TITLE

1.1 Disclaimers and usage instructions

This is a version of my PhD dissertation which I defended and got through Graduate College format review in spring 2010. I hope it works for you. Please keep in mind, however, that style policies may change. Of course, please consult the style information you received from the Graduate College.

I should acknowledge that I didn't write this from scratch. I got this template from I forget where — somewhere in LPL, I think — and modified to my taste, and to match current formatting requirements. So, this is a current snapshot of collectively developed dissertation style for the University of Arizona.

How to use this template:

- Search around in `stu-dent-dis.tex`, `ua-thesis.cls`, `disack.tex`, etc. for the names “Stuart Dent” and “Anna Mehmbuhr”, etc. Change these to your name and the names of your committee members. Also change the title of your dissertation.
- In those same files, look for anything else which needs changing.
- In the first line of `stu-dent-dis.tex`, you can set the style to `draft` or `final`.
- The shell scripts `./dbuild` and `./pbuid`, in the same directory as this file, can be used to create DVI and PDF files, respectively.
- Any \LaTeX problem you have is most likely resolvable by a Google search.
- Good luck, and have fun!

— John “Stu Dent” Kerl, April 21, 2010.

x	$g(x)$
0	0
1	1
2	8
3	27
4	64
5	125

TABLE 1.2. Here is another caption.

jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs.

Compare table 1.2 with table 1.1 which was on page 11. We conjecture, but are unable to prove, a relationship between these tabular data and the function of equation (1.3.1):

$$y = x^4 - x^2 + 1. \tag{1.3.1}$$

CHAPTER 2

CHAPTER TITLE

The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs.

Please see figure 2.1.

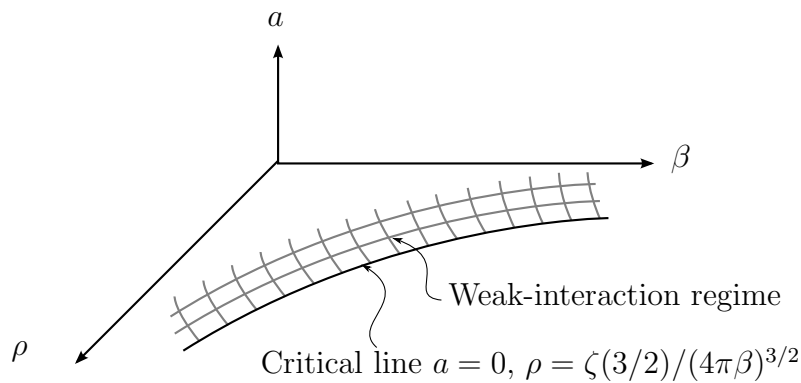


FIGURE 2.1. Here is an EPS file which I put in here and which surely must be important.

2.1 Section title

The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs. The quick brown fox jumped over the lazy dogs.

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