Exam #2 study guide · Math 124 · Calculus I · Section 26 · Fall 2008

Disclaimers about the study guide:

- Exam 2 covers sections 2.2 through 3.5. While all *topics* on the exam will be taken from this study guide, the specific *questions* on the exam will not be identical to the ones you see here.
- In addition to consulting this guide, please review all homework problems for sections 2.2 through 3.5. In particular, look at unassigned problems nearby. For example, if I assigned #14, see if you can do #13 and #15.
- For reference, you can: (*) use the back of the book; (*) use the student solution manual; (*) make use of the tutor center in Math East 145; (*) ask questions in class; (*) talk to me after class, or during office hours.

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Topics:

- Differentiability at a point $x = c$: As you take secant lines through that point c and neighboring points getting close to c, from either side, do their slopes approach some value? If so, the function is differentiable there.

  Reasons for non-differentiability at a point, given a graph:
  - The original function isn't defined at that point.
  - Discontinuity at that point.
  - A corner at that point.
  - A vertical tangent line at that point.

  Standard example: $f(x) = |x|$ at $x = 0$. Secant lines through 0 and any point to the right have slope $+1$; secant lines through 0 and any point to the left have slope -1. So, the secant lines aren't coming to any consensus as to what the slope of the derivative ought to be. (Graphically, $|x|$ has a corner at $x = 0$.)

  Another example: $f(x) = \sin(x)/x$. This looks nice and level at $x = 0$, so you'd think $f'(0) = 0$, but this function isn't defined at 0 so we don't allow ourselves to define its derivative there either.
- Numerically approximate a difference quotient:

$$\frac{f(x+h) - f(x)}{h}$$

  for small h.

  Example: $f(x) = x^2$ at $x = 3$. (For this function, we know $f'(x) = 2x$ so $f'(3) = 6$. But you can do this for all sorts of functions, even if you don't know a formula for the derivative.)

  $h = 0.1$: $[f(x+h) - f(x)]/h = (3.1^2 - 3^2)/0.1 = (9.61 - 9)/0.1 = 6.1$.

  $h = 0.01$: $[f(x+h) - f(x)]/h = (3.01^2 - 3^2)/0.01 = (9.0601 - 9)/0.01 = 6.01$.

  $h = 0.001$: $[f(x+h) - f(x)]/h = (3.001^2 - 3^2)/0.001 = (9.006001 - 9)/0.001 = 6.001$.
- Interpretations of the derivative:
  - $f' > 0$ when f is increasing
  - $f' < 0$ when f is decreasing
  - $f' = 0$ when f is constant
  - $f'' > 0$ when f is concave up
  - $f'' < 0$ when f is concave down
  - $f'' = 0$ when f is linear
  - Units of f, f', and f'': If x is in feet and $f(x)$ is in seconds, then $f'(x)$ is in feet per second and $f''(x)$ is in feet per second per second (which we also call feet per second squared).
  - Verbal interpretation of f, f', and f''.
- Sum rule: $(f + g)' = f' + g'$.

- Difference rule: $(f-g)' = f' - g'$.
- Power rule: $d/dx(x^r) = rx^{r-1}$.
- Exponential rule: $d/dx(a^x) = \ln(a)a^x$.
- Product rule: $(fg)' = f'g + fg'$.
- Quotient rule: $(f/g)' = (f'g - fg')/g^2$.
- Chain rule:
  - $f(g(x))' = f'(g(x))g'(x)$.
  - outer′(inner)inner′.
  - $(d(\text{outer})/d\Box) \cdot (d\Box/dx)$.
- Trig rules:

$$\begin{aligned} \frac{d}{dx}(\sin(x)) &= \cos(x) \\ \frac{d}{dx}(\cos(x)) &= -\sin(x) \\ \frac{d}{dx}(\tan(x)) &= \frac{1}{\cos^2(x)} \end{aligned}$$

- Be able to use multiple rules on a problem when necessary.
- Know how to use derivatives to find equations of tangent lines (use point-slope form).
  - Remember point-slope form: $y = m(x-a) + b$. This is the equation of a line with slope m going through the point (a, b).
  - Example: $f(x) = x^2$ at $x = 3$. We have $a = 3$ and $b = f(a) = 3^2 = 9$. Since $f'(x) = 2x$, $f'(3) = 6$ i.e. the slope at $x = 3$ is 6. (You have to evaluate $f(a)$ to get b, *and* you have to evaluate $f'(a)$ to get m.) Plugging these numbers into point-slope form gives

    $$y = 6(x-3) + 9 = 6x - 18 + 9$$

    which simplifies to

    $$y = 6x - 9.$$
  - If you prefer, you might wish to memorize the following:

    $$y = f(a) + f'(a)(x-a).$$

    This works since we use $f'(a)$ for m and $f(a)$ for b.
  - Check: Graph $y = x^2$ and $y = 6x - 9$ in your calculator; make sure the line is tangent to the curve at $x = 3$.
- Given a formula for $f(x)$, know how to find $f'(c)$ for specific c, e.g. $f'(5)$. This takes two steps: (1) find a formula for $f'(x)$; (2) evaluate that at $x = 5$.
- Given a graph for $f(x)$, know how to find $f'(c)$ for specific c, e.g. $f'(5)$.
- What I may ask you about second derivatives:
  - Given a function $f(x)$, find its second derivative. Just compute its first derivative $f'(x)$, then differentiate *that*. Example: $f(x) = x^3$; $f'(x) = 3x^2$; $f''(x) = 6x$.
  - Evaluate the second derivative at a point, e.g. $x = 5$. Differentiate twice to get $f''(x)$, then evaluate that at $x = 5$ to get $f''(5)$. Example: if $f(x) = x^3$ then $f''(5) = 30$.
  - Find when the original function $f(x)$ is concave up or down. Just find $f''(x)$, then solve the inequality $f''(x) > 0$ or $f''(x) < 0$, respectively. Example: $f(x) = x^3$. Then $f'(x) = 3x^2$ and $f''(x) = 6x$. The original function $f(x)$ is concave up when $f''(x) > 0$. So, solve $6x > 0$: $x > 0$. Also graph to check.