Exam #2 · Thursday, March 1, 2007

MATH 124 · Calculus I · Section 8 · Spring 2007   Name _____

Problem 1. Let $f(x) = x^x$. Numerically approximate $f'(2)$ using difference quotients. Use at least three successively smaller values of h.

Problem 2. On a mountain-climbing expedition, you find that the air becomes cooler as you climb. Let y be your altitude above sea level, measured in feet; let H be the air temperature in degrees Fahrenheit.

Part (a). What are the units of $H'(y)$?

Part (b). What is the sign of $H'(y)$?

Part (c). Give a practical interpretation of $H^{-1}(35)$.

Part (d). Give a practical interpretation of $H'(7500)$.

Problem 3. In a lab experiment, you have microorganisms growing in a Petri dish. The number m of microrganisms, in millions, is a function of time t in days since the start of the experiment. This number is given by
$$m(t) = 4.1e^{0.24t}.$$
Find the rate of change in population on day 5 of the experiment. In your answer, please show units.

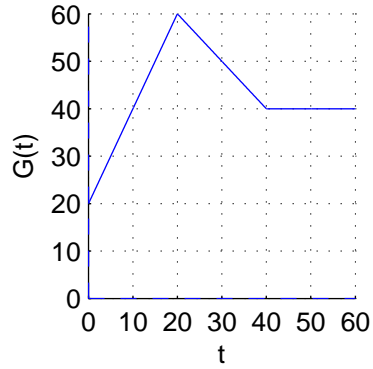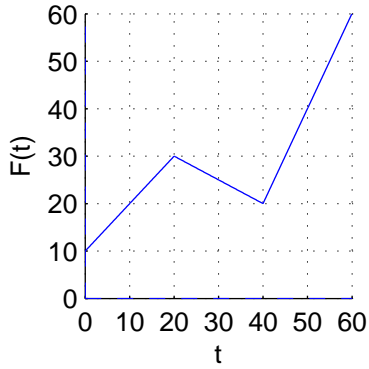**Problem 4.** Let $G(t) = 2^{-rt} \sin(at)$. Find $G'(t)$.

Problem 5. Let $f(z) = \tan^{-1}(z)$. Compute $f''(1)$.

Problem 6. Let $q(x) = \ln(2 + 2x + x^2)$.

Part (a). Find $q'(x)$.

Part (b). Find an equation for the tangent line to $q(x)$ at $x = 3$.

Problem 7. Let $F(t)$ and $G(t)$ be given by the following graphs.

Part (a). Find $G'(40)$.

Part (b). Let $H(t) = \frac{F(t)}{G(t)}$. Find $H'(50)$.

Part (c). Let $H(t) = F(G(t))$. Find $H'(30)$.

Problem 8. Let

$$f(x) = \begin{cases} \sin(x), & x \geq 0 \\ x - x^3, & x \leq 0. \end{cases}$$

Part (a). Find $f'(x)$. Write it as a piecewise function.

Part (b). Is the original function $f(x)$ differentiable at $x = 0$? Why or why not? (Hint: graph it.)

Problem 9. When is $g(x) = x^3 + bx^2 + cx + d$ concave up? Assume b, c, d are constants. (You will need to solve an inequality.)