Exam #4 · Thurday, April 26, 2007

MATH 124 · Calculus I · Section 8 · Spring 2007 Name _____

Problem 1. Let

$$\begin{aligned} x(t) &= 3t^2 - 6t \\ y(t) &= \frac{4}{3}t^3 - 4t. \end{aligned}$$

Part (a). Find the time(s) t, if any, when the particle comes to a stop.

Part (b). Find an equation for the tangent line to this curve at $t = 3$.

Problem 2. The function $H(t)$ describes the growth rate in thousands per month of flour beetles in a jar, where t is measured in months since the start of the year.

Part (a). What are the units of $\int_4^7 H(t)dt$?

Part (b). Give a practical interpretation of $\int_4^7 H(t)dt$.

Problem 3. The air pressure within a chamber is given by

$$P(t) = 2.1 + 0.4t^{0.5}$$

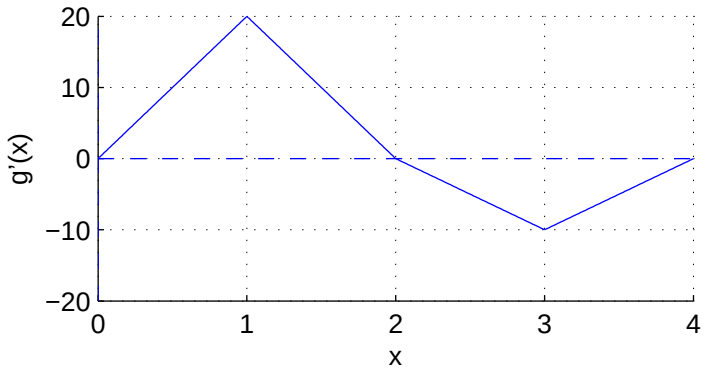
where P is in units called atmospheres and t is measured in hours. Find the average pressure over the time between $t = 2$ hours and $t = 5$ hours.

Problem 4. Find the exact area between $f(x) = e^x - 2$ and $g(x) = -1$ on the interval $[2, 4]$.

Problem 5. Find the general antiderivative:

$$\int \left(\frac{y^{2.1}}{3} - \frac{7}{y} + 0.2Ae^y + B \right) \, dy.$$

Problem 6. Let $g'(x)$ be given by the following graph, and suppose $g(0) = 2$:



Part (a). What are the x-coordinates of the critical points of $g(x)$?

Part (b). What are the x-coordinates of the inflection points of $g(x)$?

Part (c). Find the values of $g(x)$ at the critical and inflection points.

Part (d). Sketch a graph of $g(x)$. Label critical points and inflection points of $g(x)$.

Problem 7. The quantity A varies with time as specified by

$$\frac{dA}{dt} = 7.3\cos(t) - 0.04.$$

Part (a). Write down a general solution for A.

Part (b). Given that $A(0) = 2.1$, write down a specific solution for A.

Problem 8.
Part (a). Let

$$G(x) = \int_1^x e^{t^2}\, dt.$$

Determine (with justification) whether $G(x)$ is increasing, decreasing, or constant.

Part (b). Now let

$$G(x) = \int_1^{ax+b} e^{t^2}\, dt.$$

Find $G'(x)$.