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References

The random-cycle model:

[BU07] Betz, V. and Ueltschi, D. *Spatial random permutations and infinite cycles.* `arXiv:0711.1188`. Commun. Math. Phys. 285, 469-501 (2009).

[BU08] Betz, V. and Ueltschi, D. *Spatial random permutations with small cycle weights.* `arXiv:0812.0569v1`. Probabl. Th. Rel. Fields (2010).

[BU10] Betz, V. and Ueltschi, D. *Critical temperature of dilute Bose gases.* Physical Review A **91**, 023611 (2010). `arXiv:0910.3558`.

[BUV09] Betz, V. Ueltschi, D., and Velenik, Y. *Random permutations with cycle weights.* `arXiv:0908.2217`.

[GRU] Gandolfo, D., Ruiz, J., and Ueltschi, D. *On a model of random cycles.* `arXiv:cond-mat/0703315`. Statist. Phys. 129, 663-676 (2007).

[Lugo] Lugo, M. *Profiles of permutations.* Electronic Journal of Combinatorics, **16** (2009) R99.

[Sütő1] Sütő, A. *Percolation transition in the Bose gas.* J. Phys. A: Math. Gen. **26** (1993) 4689-4710.

[Sütő2] Sütő, A. *Percolation transition in the Bose gas II.* J. Phys. A: Math. Gen. **35** (2002) 6995-7002.

[U06] Ueltschi, D. *Feynman cycles in the Bose gas.* `arXiv:math-ph/0605002v2`.

[U07] Ueltschi, D. *The model of interacting spatial permutations and its relation to the Bose gas.* `arXiv:0712.2443v3`. Mathematical Results in Quantum Mechanics, pp. 225-272, World Scientific (2008).

Non-spatial permutations:

[DF] D.S. Dummit and R.M. Foote. *Abstract Algebra* (2nd ed.). John Wiley and Sons, 1999.

[Golomb] Golomb, S.W. *Random permutations.* Bull. Amer. Math. Soc. 70 (1964), 747.

[SL] Shepp, L.A. and Lloyd, S.P. *Ordered Cycle Length in a Random Permutation.* Trans. Amer. Math. Soc. 121, (1966), 340-357.

[Ewens] Ewens, W.J. *The sampling theory of selectively neutral alleles*. Theor. Popul. Biol. 3, 87-112 (1972).

Bose-Einstein condensation:

[AEMWC] Anderson, M.H., Ensher, J.R., Matthews, M.R., Wieman, C.E., and Cornell, E.A. *Observation of Bose-Einstein Condensation in a Dilute Atomic Vapor. Science*, vol. 269, issue 5221 (Jul. 14 1995), 198-201.

[BBHLV] Baym, G., Blaizot, J.-P., Holzmann, M., Laloë, F., and Vautherin, D. *Bose-Einstein transition in a dilute interacting gas*. `arXiv:cond-mat/0107129v2`.

[AM] Arnold, X. and Moore, X. *BEC transition temperature of a dilute homogeneous imperfect Bose gas*. Phys. Rev. Lett. vol. 87, no. 12, Sept. 2001.

[BP] Buffet, E. and Pulè, J.V. *Fluctuation properties of the imperfect Bose gas*. J. Math. Phys. vol. 24, no. 6, June 1983.

[Ceperley] Ceperley, D.M. *Path integrals in the theory of condensed helium*. Rev. Mod. Phys. vol. 67, no. 2, April 1995.

[Feynman] Feynman, R.P. *Atomic Theory of the λ Transition in Helium*. The Physical Review, vol. 91, no. 6 (1953).

[Grüter] Grüter, P. *Contribution à la Théorie des Gaz Dilués-Dégénérés*. Doctoral dissertation, Université de Paris Sud, Département de Physique de l'École Normale Supérieure, 1996.

[Holzmann] Holzmann, M. *La Transition de Bose-Einstein dans un gaz dilué*. Doctoral dissertation, Université de Paris VI, Département de Physique de l'École Normale Supérieure, 2000.

[LLS] Lebowitz, J., Lenci, M., and Spohn, H. *Large deviations for ideal quantum systems.* J. Math. Phys. 41, 1224-1243 (2000).

[LSSY] Lieb, E.H., Seiringer, R., Solovej, J.P., and Yngvason, J. *The Mathematics of the Bose Gas and its Condensation*. Oberwolfach Seminars, vol. 34. Birkhäuser, 2005.

[London] London, F. Phys. Rev. vol. 54, no. 947 (1938).

[PO] Penrose, O. and Onsager, L. *Bose-Einstein Condensation and Liquid Helium*. The Physical Review, vol. 104, no. 3 (1956).

[SU09] Seiringer, R. and Ueltschi, D. *Rigorous upper bound on the critical temperature of dilute Bose gases*. `arXiv.org:0904.0050`. Phys. Rev. B 80, 014502 (2009).

Worm algorithm for path-integral Monte Carlo:

[BPS06] Boninsegni, M., Prokof'ev, N.V., and Svistunov, B.V. *Worm algorithm and diagrammatic Monte Carlo: A new approach to continuous-space path integral Monte Carlo simulations.* Physical Review E **74**, 036701 (2006).

[PST98] Prokof'ev, N.V., Svistunov, B.V., and Tupitsyn, I.S. *Exact, complete, and universal continuous-time worldline Monte Carlo approach to the statistics of discrete quantum systems.* Journal of Experimental and Theoretical Physics, vol. 87, no. 2 (1998).

Finite-size scaling for path-integral Monte Carlo:

[Barber] Barber, M.N. *Finite-size scaling.* Phase Transitions and Critical Phenomena, Vol. 8., pp 146-266. Academic Press, London, 1983.

[GCL97] Grüter, P., Ceperley, D., and Lalöe, F. *Critical temperature of Bose-Einstein condensation of hard-sphere gases.* Physical Review Letters, vol. 79, no. 19 (1997).

[HK99] Holzmann,M. and Krauth, W. *Transition temperature of the homogeneous, weakly interacting Bose gas.* Physical Review Letters, vol. 83, no. 14 (1999).

[KPS] Kashurnikov, V.A., Prokof'ev, N.V., and Svistunov, B.V. *Critical temperature shift in weakly interacting Bose gas.* Physical Review Letters, vol. 87, no. 12 (2001).

[LB] Landau, D.P. and Binder, K. *A Guide to Monte Carlo Simulations in Statistical Physics* (2nd ed.). Cambridge University Press, 2005.

[NL04] Nho, K. and Landau, D.P. *Bose-Einstein condensation temperature of a homogeneous weakly interacting Bose gas: Path integral Monte Carlo study.* Physical Review A **70**, 053614 (2004)

[PC87] Pollock, E.L. and Ceperley, D.M. *Path-integral computation of superfluid densities.* Physical Review B, vol. 36, no. 16 (1987).

[PGP08] Pilati, S., Giorgini, S., and Prokof'ev, N.V. *Critical temperature of interacting Bose gases in two and three dimensions.* Physical Review Letters **100**, 140405 (2008).

[PR92] Pollock, E.L. and Runge, K.J. *Finite-size-scaling analysis of a simulation of the ^{4}He superfluid transition.* Physical Review B, vol. 46, no. 6 (1992).

[PV] Pelissetto, A. and Vicari, E. *Critical Phenomena and Renormalization-Group Theory.* Physics Reports, 368 (6), p.549-727, Oct 2002. `arxiv:cond-mat/0012164`.

Quantum mechanics and statistical mechanics:

[Griffiths] Griffiths, D.J. *Introduction to Quantum Mechanics* (2nd ed.). Pearson Prentice Hall, 2005.

[Huang] Huang, K. *Introduction to Statistical Physics*. CRC Press, 2001.

[Sakurai] Sakurai, J.J. *Modern Quantum Mechanics* (2nd ed.). Addison Wesley, 1993.

Probability, stochastic processes, and statistics:

[Berg] Berg, B. *Markov Chain Monte Carlo Simulations and Their Statistical Analysis*. World Scientific Publishing, 2004.

[CB] Casella, G. and Berger, R.L. *Statistical Inference* (2nd ed.). Duxbury Press, 2001.

[FG] Fristedt, B. and Gray, L. *A Modern Approach to Probability Theory*. Birkhäuser, 1997.

[GS] Grimmett, G. and Stirzaker, D. *Probability and Random Processes*, 3rd ed. Oxford, 2001.

[Lawler] Lawler, G. *Introduction to Stochastic Processes* (2nd ed.). Chapman & Hall/CRC, 2006.

[Øksendal] Øksendal, B. *Stochastic Differential Equations* (6th ed.). Springer, 2007.

[Young] Young, H.D. *Statistical Treatment of Experimental Data*. McGraw-Hill, 1962.

Algorithms:

[MN] Matsumoto, M. and Nishimura, T. Mersenne twister: a 623-dimensionally equidistributed uniform pseudo-random number generator. ACM Transactions on Modeling and Computer Simulation, Volume 8, Issue 1 (Jan. 1998), pp 3-30.

[NR] Press, W. et al. *Numerical Recipes* (2nd ed.). Cambridge, 1992.