Daniel:

In trying to derive the expression for the shift in critical temperature for the two-cycle model, I think I am missing some physics.

I start with (5.18):

$$\rho_c^{(\alpha)} = \rho_c^{(0)} - \frac{1 - e^{-2\alpha}}{(8\pi\beta)^{3/2}}.$$

Expanding to first order in α:

$$\rho_c^{(\alpha)} \approx \rho_c^{(0)} - \frac{2\alpha}{(8\pi\beta)^{3/2}}.$$

Using (5.7), namely, $\alpha = (8/\pi\beta)^{1/2}a$:

$$\begin{aligned}\rho_c^{(a)} &\approx \rho_c^{(0)} - \frac{2 \cdot 8^{1/2} \cdot a}{\pi^{1/2} \cdot \beta^{1/2} \cdot 8^{3/2} \cdot \pi^{3/2} \cdot \beta^{3/2}} \\ &= \rho_c^{(0)} - \frac{a}{4\pi^2\beta^2}.\end{aligned}$$

Using $\beta = 1/T_c^{(0)}$:

$$\rho_c^{(a)} = \rho_c^{(0)} - \frac{{T_c^{(0)}}^2 a}{4\pi^2}.$$

Using the near-critical approximation $\rho \sim T^{3/2}$ from the old version of your 2007 paper with Betz:

$$\begin{aligned}{T_c^{(a)}}^{3/2} &= {T_c^{(0)}}^{3/2} - \frac{{T_c^{(0)}}^2 a}{4\pi^2} \\ \left(\frac{T_c^{(a)}}{T_c^{(0)}}\right) &= 1 - \frac{{T_c^{(0)}}^{1/2} a}{4\pi^2} \\ \frac{T_c^{(a)}}{T_c^{(0)}} &= \left(1 - \frac{{T_c^{(0)}}^{1/2} a}{4\pi^2}\right)^{2/3}.\end{aligned}$$

Taylor-expanding the right-hand side in the small parameter a:

$$\begin{aligned}\frac{T_c^{(a)}}{T_c^{(0)}} &= 1 - \frac{{T_c^{(0)}}^{1/2} a}{6\pi^2} \\ \frac{T_c^{(a)}}{T_c^{(0)}} - 1 &= -\frac{{T_c^{(0)}}^{1/2} a}{6\pi^2} \\ \frac{T_c^{(a)} - T_c^{(0)}}{T_c^{(0)}} &= -\frac{{T_c^{(0)}}^{1/2} a}{6\pi^2}.\end{aligned}$$

Again using the near-critical approximation $T^{1/2} \sim \rho^{1/3}$:

$$\frac{T_c^{(a)} - T_c^{(0)}}{T_c^{(0)}} = -\frac{\rho^{1/3} a}{6\pi^2}.$$

The minus sign does not belong there, but I don't see the error in my work.

Questions:

- Why is it true that $\rho \sim T^{3/2}$ near T_c? (The original statement in your 2007 paper with Betz was that $\beta \sim \rho^{-2/3}$ near $T_c(0)$.)
- What is the scale factor connecting ρ and $T^{3/2}$: is it 1?
- Even though $\rho \sim T^{3/2}$, can one conclude that necessarily $\rho_c \sim T_c^{3/2}$?
- I cannot reproduce the factor of 0.74 in (5.19). If in fact $\rho \sim KT^{3/2}$ for some scale factor K other than 1, I get

$$\frac{T_c^{(a)} - T_c^{(0)}}{T_c^{(0)}} = -\frac{\rho^{1/3} a}{6K^2\pi^2}$$

so I would hope that $1/6K^2\pi^2 \approx 0.74$.

Thank you!