

Exam #4 · Fri. Dec. 2, 2005

MATH 110 · Section 10 · Fall 2005

Name \_\_\_\_\_

**REMINDER:** *The final exam is Monday, December 12, 8:00 a.m. to 10:00 a.m., in CESL 103. Please arrive at 7:40 a.m.*

Formulas:

$$\begin{array}{lll} A = Pe^{rt} & \sum_{k=1}^n k = \frac{n(n+1)}{2} & \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6} \\ a_n = a_1 + (n-1)d & S_n = n \left( \frac{a_1 + a_n}{2} \right) & S_n = \frac{n}{2} (2a_1 + (n-1)d) \\ a_n = a_1 r^{n-1} & S_n = a_1 \left( \frac{1-r^n}{1-r} \right), r \neq 1 & \sum_{k=1}^{\infty} a_1 r^{k-1} = \frac{a_1}{1-r}, |r| < 1 \end{array}$$

**Problem 1. (6 points)** One may approximate the altitude  $A$  (in meters above sea level) using the formula

$$A = 22,860 \ln \left( \frac{p_0}{p} \right)$$

where  $p$  is the atmospheric pressure in mm Hg at that altitude, and  $p_0$  is the sea-level atmospheric pressure, 760 mm Hg. At an altitude of 4,000 meters, what is the approximate atmospheric pressure?

**Problem 2. (6 points)** Your aunt Judy gave you a generous high-school graduation gift of \$4,000, which you wisely invested in an account yielding 6% annual interest (compounded continuously). How long (in years, to the nearest 0.1 year) will it take for the balance to reach \$10,000?

**Problem 3. (6 points)** Consider the sequence whose terms given by

$$a_n = \frac{(-1)^n}{n^2 + 1}.$$

Which of the following statements are true about the 6th and 7th terms of the sequence?

$$(1)a_6 < 0 \quad (2)a_7 < 0 \quad (3)a_6 < a_7$$

(A) 1, 2, and 3 (B) 1 and 2 only (C) 1 only (D) 2 only (E) None of these

**Problem 4.** Consider the sequence

$$\frac{3}{2}, \frac{4}{4}, \frac{5}{6}, \frac{6}{8}, \frac{7}{10}, \dots$$

(a) **(4 points)** Find a closed-form expression for  $a_n$ . (This means that the formula for  $a_n$  should not refer to  $a_{n-1}$ .)

(b) **(4 points)** What is the 20th term of the sequence?

(c) **(4 points)** Describe the long-term behavior of the sequence: does it approach a specific value? If so, what and why? If not, why not?

**Problem 5. (5 points)** Which of the following sequences are arithmetic?

(1)  $1, -1, 1, -1, 1, \dots$       (2)  $2, 2, 2, 2, 2, \dots$       (3)  $1, 4, 9, 16, 25, \dots$

(A) 2 only    (B) 3 only    (C) 2 and 3 only    (D) 1 and 2 only    (E) None of these

**Problem 6. (5 points)** Which of the following sequences are geometric?

(1)  $1, -1, 1, -1, 1, \dots$       (2)  $2, 2, 2, 2, 2, \dots$       (3)  $1, 4, 9, 16, 25, \dots$

(A) 2 only    (B) 3 only    (C) 2 and 3 only    (D) 1 and 2 only    (E) None of these

**Problem 7. (6 points)** Given that

$$\sum_{k=1}^{10} ck = 33,$$

determine the value of  $c$ .

**Problem 8. (6 points)** Evaluate the sum

$$\sum_{k=4}^{20} 7k^2.$$

**Problem 9. (6 points)** Evaluate the sum

$$\sum_{k=1}^8 4 \left(\frac{1}{3}\right)^{k-1}.$$

**Problem 10. (6 points)** Evaluate the sum

$$\sum_{k=1}^{\infty} 4 \left(\frac{1}{3}\right)^{k-1}.$$

**Problem 11. (6 points)** Find a closed-form expression for the  $n$ th term of the arithmetic sequence with first term 7.2 and common difference 0.1.

**Problem 12. (6 points)** Find the number of terms in the sequence

$$11.3, 11.5, 11.7, \dots, 57.5.$$

**Problem 13. (6 points)** Find the sum of the even positive integers up to and including 500.

**Problem 14.** An employer in Connecticut has offered you a starting salary of \$40,000 per year with 3% annual increases. An employer in San Diego has offered you a starting salary of \$35,000 per year with 4% annual increases.

(a) **(6 points)** For the Connecticut job, what would be your total earnings over a 10-year period?

(b) **(6 points)** For the San Diego job, what would be your total earnings over a 10-year period?

**Problem 15. (6 points)** As you rise through management, you will need to deal with other people's annoying problems more and more often. If you deal with 8 annoyances per month in your entry-level position, and if the number of annoyances per month increases by 20 percent for each level you rise in the company, how many annoyances per month can you expect once you reach regional vice president, which is the seventh layer of management?