Exam #1 · Friday, February 2, 2007

MATH 124 · Calculus I · Section 8 · Spring 2007 Name _____

Problem 1. Let $f(x) = A(x - B)^2(x + C)$ with $A > 0$, $B > 0$, and $C > 0$. Using the following space, sketch a graph of $f(x)$. Include and label the x-intercept(s), if any, and the y-intercept. (The x-intercepts and y-intercept should be in terms of A, B, and C — please do not plug in specific numerical values for A, B, and C.)

Problem 2. Sally's track coach tells her that to avoid injury, she should increase her weekly mileage by 10% per week.

Part (a). If she currently runs 2 miles per week, and if she follows her coach's advice, by how many weeks from now will she be running 25 miles per week?

Part (b). If she currently runs 2 miles per week, and if she instead follows her friend's advice and increases her mileage by 1.5 miles per week, by how many weeks from now will she be running 25 miles per week?

Problem 3. Let

$$f(z) = \frac{(z-1)(z-2)}{(z-3)(z-4)}.$$

Part (a). Find the domain of $f(z)$.

Part (b). Find the zeroes of $f(z)$.

Part (c). Find the vertical asymptote(s), if any, of $f(z)$.

Part (d). Find the horizontal asymptote(s), if any, of $f(z)$.

Part (e). Find the hole(s), if any, of $f(z)$.

Problem 4. In your physics class, you attach an oscillosope to a piece of electronic equipment and discover that the voltage you observe looks like a sine wave. In particular, you notice that

- The maximum voltage you observe is 1.8 volts.
- The minimum voltage you observe is 0.0 volts.
- This maximum voltage is found at the start of your data (i.e. $t = 0.0$ picoseconds); the first time it reoccurs is 2.0 picoseconds later (i.e. $t = 2.0$ picoseconds).

Find an equation of the form $a + b\cos(ct)$ that describes the voltage V as a function of time t in picoseconds. (You don't need to know how many picoseconds make a second.) (Hint: You might want to sketch a graph.)

Problem 5. In a lab experiment, you have microorganisms living in a petri dish. Assume that the microorganisms are dying at an exponential rate. You have the following data:

- You forgot to count the initial population.
- You count a population of 2500 organisms on day one.
- You count a population of 1500 organisms on day two.

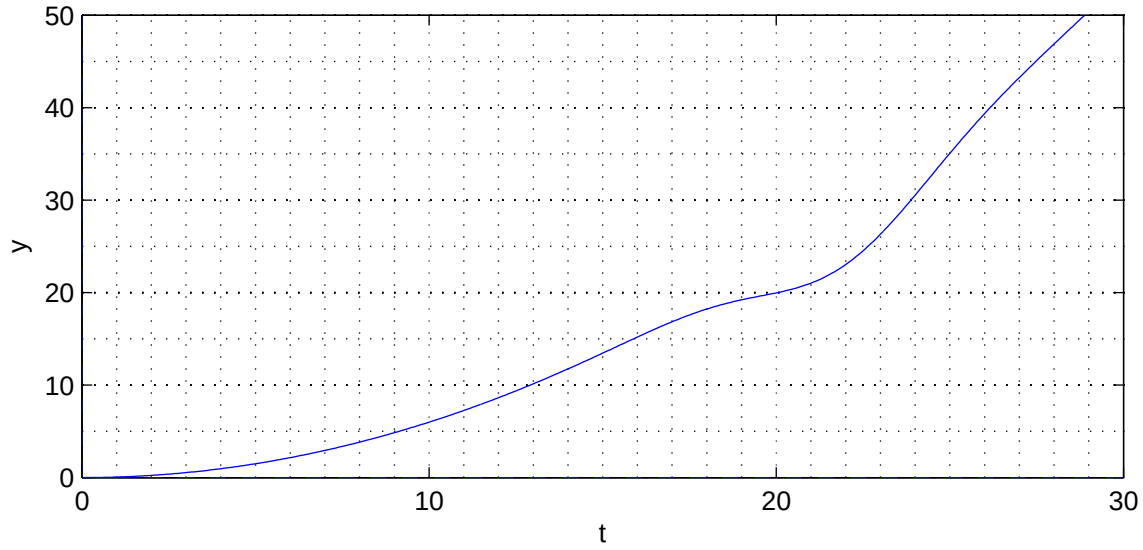
Find an exponential model of the form $y = Ce^{rt}$ for these data, where y is the number of microorganisms in the dish after t days. That is, find C and r so this model matches the given data.

Problem 6. You leave on a car trip for San Diego. The following events happen:

(i) You drive at medium speed (in town) for the first 15 minutes.
(ii) Once you get out of town, you accelerate almost immediately to the speed limit and drive at that speed for 45 minutes.
(iii) Then, you hit a traffic jam. You come to a stop almost immediately and sit at a stop for 30 minutes.
(iv) Once past the accident, you accelerate almost immediately back to the speed limit and drive at that speed until you reach the coast 4 hours later.

Sketch a graph of distance D from your house vs. time t in hours. Label all the above events on the t axis.

Problem 7. The following graph shows the altitude y of a rocket (in miles) as a function of time t since launch (in seconds):



Part (a). Estimate $f(18)$, and describe verbally what it means to you. Include units in your response.

Part (b). Estimate $f'(18)$, and describe verbally what it means to you. Include units in your response.

Part (c). Estimate $f^{-1}(40)$, and describe verbally what it means to you. Include units in your response.

Problem 8. Let $f(x) = 2x^2 + 1$. Evaluate

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

using properties of limits.

Problem 9. The following are selected values of a function $f(x)$.

$f(3) =$ is undefined $\qquad f(4) = -6 \qquad f(6) = 9$

Table A	
x	$f(x)$
2.9	-8.14286
2.99	-98.01493
2.999	-998.00150
2.9999	-9998.00015

Table B	
x	$f(x)$
3.1	-12.15789
3.01	-102.01508
3.001	-1002.00150
3.0001	-10002.00015

Table C	
x	$f(x)$
3.9	-5.56566
3.99	-5.95070
3.999	-5.99501
3.9999	-5.99950

Table D	
x	$f(x)$
4.1	6.57576
4.01	6.05071
4.001	6.00501
4.0001	6.00050

Table E	
x	$f(x)$
5.9	6.32184
5.99	6.49616
5.999	6.49956
5.9999	6.49999

Table F	
x	$f(x)$
6.1	6.63196
6.01	6.59234
6.001	6.50034
6.0001	6.50002

Table G	
x	$f(x)$
100	1.05285
1000	1.00503
10000	1.00050
100000	1.00005

Table H	
x	$f(x)$
-100	2.95257
-1000	2.99503
-10000	2.99950
-100000	2.99995

Part (a). Find the following limits, when they exist. If a limit does not exist, say why not. For each, state which table(s) (A, B, C, D, E, F, G, H) you use to support your answer.

$\lim_{x\to 3} f(x) \qquad \lim_{x\to 4} f(x) \qquad \lim_{x\to 6^-} f(x) \qquad \lim_{x\to -\infty} f(x)$

Part (b). Determine if $f(x)$ is continuous or not continuous at the following values. If $f(x)$ is not continuous at the point, describe what feature occurs there.

$x = 3 \qquad x = 4 \qquad x = 6$