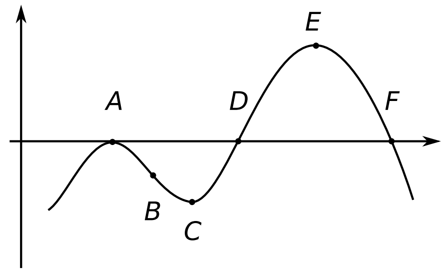
Exam #3 · Tuesday, April 3, 2007

MATH 124 · Calculus I · Section 8 · Spring 2007 Name ______________________

Problem 1. Consider the following graph of $f'(x)$:



I repeat for emphasis: I am showing you the graph of $f'(x)$, but I will ask you questions about $f(x)$.

Part (a). Which of the labeled points are critical points of $f(x)$?

Part (b). Which of those critical points are local maxima (not minima) of $f(x)$? (In this problem, I am not interested in local extrema at boundary points.)

Part (c). Which labeled points are inflection points of $f(x)$?

Problem 2. Find dy/dx if $\ln(y)\sin(y) = \cos(x)$.

Problem 3. Let $f(z) = z^4 + a/z^4$. Find a such that $f(z)$ has a minimum at $z = 2$.

Problem 4. Let $g(x) = \sin^2(x) - \cos(x)$.

Part (a). Find an equation for the tangent line to $g(x)$ at $x = \pi/2$.

Part (b). Write down an equation for the error function involving $g(x)$ and the linear approximation.

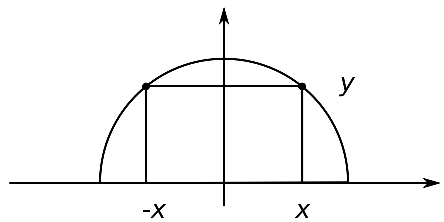
Problem 5. Find b and d such that $f(x) = x^3 + bx^2 + d$ has an inflection point at $x = 3$ and y-intercept -5.

Problem 6. Does the limit

$$\lim_{x\to\infty} \frac{x^2}{e^{0.0024x}}$$

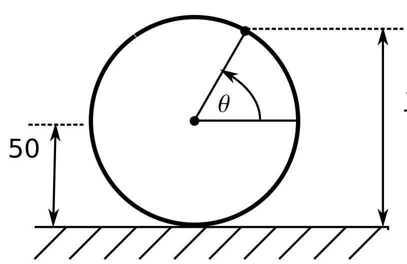
exist? If so, what is it, and why? If not, why not?

Problem 7. A rectangular region is to be painted inside a semicircular region as follows:



The semicircle has radius 7 feet. Find x that maximizes the area of the rectangle. (You must show all your work and use calculus.)

Problem 8. An amusement-park ride is upright and circular with 50-foot radius. Passengers are seated around the perimeter while the wheel turns counterclockwise at some (as yet unknown) constant rate:



The height of the labeled passenger above the ground is given by

$$y = 50 + 50\sin(\theta).$$

If the passenger's height above ground is increasing by 3.2 feet per second at angle $\theta = \pi/3$, how fast is the wheel turning? Include units in your answer.